Data Science

5. Exercise / Practical

Theoretical Exercises

Excercise 5.1: (Theoretical) Throwing a fair coin

A fair coin is thrown three times. We denote the result with e.g. HHT for head, head, tail. We consider the random variable X: "number of head".

a) What is the probability distribution of X? Give it in form of a probability density.

b) What are the expected value, variance and standard deviation of X?

c) How large is the probability that head occurs at least once?

Excercise 5.2: (Theoretical) Thin wire

A current flows through a thin wire for which it is known that its strength X (in mA) is uniformly distributed in the interval [2, 6].

a) Specify the density f(x) and the expected value E(X).

b) With is the probability that a measurement show that more than 4 mA is flowing?

Excercise 5.3: (Theoretical) Premature babies

A study on the health status of premature babies is being carried out in a clinic. The birth weight X of a child born in the 28th week of pregnancy is assumed to be a normally distributed random variable with an expected value of 1000 g and a standard deviation of 50 g. g.

a) What is the probability that a child born in the 28th week of pregnancy has a weight between 982 and 1050 g?

b) Determine the 10% quantile of the birth weight. What does it say?

Excercise 5.4: (Theoretical) Poisson distribution

During the production of optical storage media, disturbing dust particles occur. It is assumed that the number of dust particles follows a Poisson distribution, with an average of 5 dust particles per 100 cm^2.

a) What is the probability of finding less than 3 dust particles on a CD of 100 cm^2?

b) How many dust particles can be expected on a CD of 100 cm^2 with what standard deviation?

$Poi(\lambda)$-distributed

A discrete random variable X with support $\mathbb{N}_0$ has a **Poisson distribution** ($X \sim Poi(\lambda)$) with parameter $\lambda \in \mathbb{R}^+$, if the probability density function is given by

$$f(x) = \begin{cases} \exp(-\lambda)\frac{\lambda^x}{x!}, & x = 0, 1, 2, \dots \\ 0, & otherwise \end{cases}$$

Practical Exercises

Excercise 5.5: (Practical) Sampling values

Create a vector of length n where each value is sampled from a normal distribution with arbitrary μ and σ.

1. Plot the histogram for different values of n, compare it with the density function.
2. Calculate the mean and variance of the vector.
3. Compare the value with the expected value and the variance of the normal distribution under consideration and plot the difference for different values of n
4. Consider different values for μ and σ, how do the plots differ?

Hint: Sampling values from a normal distribution can be done with the scipy module (scipy.stats.norm).